

EFFECTS OF SMALL VIBRATIONS ON THE DEFORMATION OF POLYMERS

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In a recent investigation [1], it was discovered that small oscillatory loads have a much stronger influence on the creep rate of stiff crystalline polymer materials, than one would expect at first glance. In the present work, it is shown that this phenomenon reveals a wide class of problems on the deformation of polymers in connection with which the influence of small vibrations exists, and may become a determining factor.

The main cause of the strong influence of small vibrations on the deformation of polymers is the fact that the polymer materials are viscoelastic and have a low thermal conductivity. Hence, even under the influence of small vibrations the polymers may heat up rapidly. The heating leads to a change in the deformation characteristics of the material, in particular to a lowering of the stiffness. This leads to an increase in deformation which, in turn, causes additional heating.

In this paper we describe a general approach to the study of the influence of small vibrations on the deformation of stiff polymers. The basic underlying assumption is that this influence arises as a result of temperature change due to vibrations and the consequent change in the deformation properties of the material. Based on this hypothesis plus some additional assumptions, two complete systems of equations are obtained: one for the average stress and strain states in a variable but known temperature field; the other for the characteristics of the 'vibration' field and the temperature. Based on experimental values of the moduli of elasticity and viscosity of the materials investigated, together with certain estimates on terms entering the dynamic equations of the second system, the viscosity and inertia terms are neglected while the dependence of the modulus of elasticity on the temperature is retained. The viscosity is taken into account only in the energy equation, where it determines the additional heating as a result of the dissipation of the energy of small vibrations. In particular, the proposed approach provides a known basis for the investigation of the influence of vibrations on the strength of polymers, as well as for the investigation of the dynamic stability of elastic equilibria of polymer structures.

1. Consider a stiff but sufficiently viscous polymer material such as caprone or

caprolite.

Under continuous influence of a slowly varying (or steady) load, these materials exhibit relaxation properties which depend essentially on temperature. Figure 1 shows creep curves for caprolite at different temperatures which are typical for such materials, thus corroborating the above noted statement.

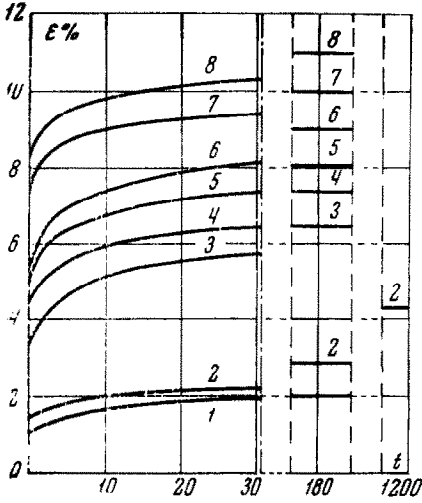


FIG. 1

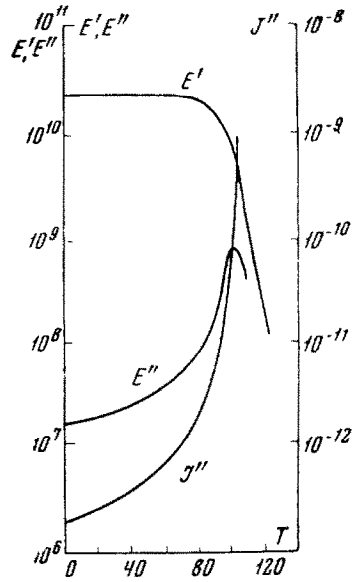


FIG. 2

Under the action of a sinusoidal oscillatory load, the materials under investigation behave as viscoelastic bodies characterized by the elastic modulus E' and viscosity η (or the frequency ω dependent modulus which we shall call the 'loss' modulus, and which is related to the viscosity by $E'' = \eta\omega$), depending on the frequency, on the temperature T , and on the Poisson's ratio ν , which is assumed to be independent of the frequency and temperature. Figure 2 shows the temperature dependence of E' and E'' taken from [2] for polystyrene, a material which is representative of the type under investigation.

An analysis of the experimental data shown leads to the following conclusions:

1) Deformation characteristics of materials of the type under investigation under the action of slowly varying loads depend essentially on the temperature.

2) For the fixed frequency oscillations, the modulus of elasticity E' depends on the temperature. It decreases relatively slowly till it reaches a certain critical temperature T_c , beyond which it decreases rapidly, and changes by several orders of magnitude over a small temperature interval. Indeed, upon reaching the critical temperature, the material loses its former structural properties.

3) Within the temperature range of practical interest, the loss modulus E'' , for a fixed frequency of oscillation, increases with the rise of temperature, first slowly, then quickly. It reaches a maximum at the same value of the critical temperature then it begins to decrease rapidly.

4) For $T < T_c$, E'' is at least an order of magnitude smaller than E' , which never decreases below 10^9 dynes/cm² or about 10^5 kg/cm². In this temperature range, the acoustic velocities in the materials under investigation are high (of the order of 10^5 cm/sec), hence in the investigation of low frequency vibrations (of the order of 100 cps and less) we may exclude from the investigation wave phenomena for objects of dimensions of the order of one meter or less and assume the stress fields to be quasistatic.

The preceding conclusions permit a considerable simplification of the investigation the influence of small vibrations on the deformations of polymer materials.

2. To illustrate the general approach, we will study a very simple example: The deformation of a prismatic bar with cross-sectional area S under the action of a compressive or tensile longitudinal force $P_0 = \sigma_0 S$. Let us assume that under the influence of static or slowly varying load the material behaves as an ideal elastic body with a temperature – dependent elastic modulus $E(T)$. For short time intervals, materials of the type examined in section 1 behave like such models (here and from now on an effective value of the elastic modulus is taken). In addition to the basic load P_0 , the bar is subjected to an oscillatory load $P^* = \sigma^* S \cos \omega t$, where the amplitude of the oscillations σ^* is much smaller than σ_0 . In view of the smallness of amplitude of the average oscillatory stresses in comparison with the average stresses due to the basic load when the averages are taken over a time interval which is large compared with the period of oscillation, but small compared with the characteristic time of temperature variation, we may write the usual equations of elasticity containing a temperature dependent elasticity modulus and possessing a trivial solution

$$\langle \sigma \rangle = \sigma_0, \quad \langle \varepsilon \rangle = \sigma_0 / E(T) \quad (2.1)$$

Here σ is the stress, ε is the strain, and the symbol $\langle \rangle$ denotes the mean value in the sense described above. In view of their quasistatic character, the longitudinal vibrations of a bar under the influence of the additional oscillatory load are given by

$$m \frac{d^2 \varepsilon'}{dt^2} + \eta \frac{d\varepsilon'}{dt} + E' \varepsilon' = \sigma^* \cos \omega t \quad (2.2)$$

Here ε' is the oscillatory strain, $m = \rho l^2 / 2$, l is the length of bar, ρ is the material density (remembering that under the action of the oscillatory load, by assumption, the material behaves as a viscoelastic substance). Noting that the characteristic time of variation of the temperature and consequently of the two moduli E' and E'' is much greater than the period of oscillation, so that the two moduli may be treated as constants, the solution of (2.2) may be written as

$$\varepsilon' = \frac{\sigma^* \cos(\omega t - \delta)}{\{[E' - m\omega^2]^2 + E''^2\}^{1/2}}, \quad \tan \delta = \frac{E''}{E' - m\omega^2} \quad (2.3)$$

At the same time, the average energy dissipation per unit volume of material per unit

time is

$$Q = \frac{\sin \delta \omega \sigma^{*2}}{2\{[E' - m\omega^2]^2 + E''^2\}^{1/2}} \quad (2.4)$$

For a bar length of the order of tens of *cm*, *m* is of the order of 10^2 g/cm; and ω is of the order not exceeding 10^3 sec⁻¹, so that, in view of the preceding discussion, $E' \gg m\omega^2$. Within the temperature range of practical interest, $E'' \ll E'$, as noted in section 1. Hence, $\tan \delta \sim \sin \delta \sim E'' / E' \ll 1$. Neglecting small quantities in (2.3) and (2.4), we obtain

$$\epsilon' = \frac{\sigma^* \cos \omega t}{E'(T)}, \quad Q = \frac{\sigma^{*2} \omega}{2} I''(T, \omega), \quad I'' = \frac{\sin \delta}{E'} = \frac{E''}{E'^2} = \frac{1}{2\pi} J'' \quad (2.5)$$

where I'' is the so-called 'yielding loss' which depends only on temperature and frequency and is independent of stress. The example leads to the following conclusions: The inertia and viscous terms in the dynamic equations for the oscillatory strains may be neglected. Within the previously shown temperature and frequency ranges, the oscillatory strains for materials of the type under investigation may be computed from the static stress-strain relations using the instantaneous value of the modulus of elasticity $E'(T)$.

This conclusion may be reached just by estimating several terms in the dynamic equation (2.2), without obtaining its solution. Indeed, the order of magnitude of the ratio of the first (inertia) and third terms in the left hand side of (2.2) is $\rho\omega^2 l^2 / E' \ll 1$, and the order of magnitude of the ratio of the second term to the third is $E'' / E' \ll 1$, whence (2.5) follows.

3. Based on the preliminary estimates of the preceding section, we will examine the general three-dimensional case. Consider a body composed of a polymer material of the type under investigation acted upon by constant (or slowly varying) distributed body forces g^m together with surface forces p^m distributed over a portion of the surface, S_1 ; on the remaining portion of the surface S_2 , the displacements U_0 are given. In addition to the preceding loading, the body is subjected to the following oscillatory loads: Body force $g^a \cos \omega t$ and surface forces $p^a \cos \omega t$. The following assumptions are made with regard to the oscillating loads:

(1) The amplitudes of the oscillating loads, g^a and p^a are small in comparison with the basic loads g^m and p^m .

(2) The periods of the oscillating loads are sufficiently large to neglect the dynamic effects and sufficiently small in comparison with the characteristic time of variation of the basic loads and temperature.

We will now go on to the derivation of the basic relations, and begin by formulating two basic hypotheses.

First hypothesis. The influence of small oscillations on the deformation process for polymers of the type under investigation depends on the change brought about in the

properties of slow deformation of the material as a result of a temperature change.

It is understood that this hypothesis narrows the class of materials and possibly frequencies to which the theory developed below applies. The experiments of Shesterikov and Lokoshchenko [3], conducted on duralumin and the experiments of Slonimskii and Alekseev, conducted on elastomers under conditions eliminating heating effects have shown, that oscillatory stresses may have an accelerating effect on the deformation process, the effect being independent of temperature. However, results published in [1] and other, similar experiments and analyses show that heating appears as the basic factor for materials of the type under investigation.

Second hypothesis. The materials under investigation behave under oscillatory stresses as viscoelastic bodies with the viscoelastic properties E' and E'' temperature dependent but not stress dependent.

The characteristics of the stress-displacement state of the body may be represented in the form

$$u_i = \langle u_i \rangle + u_i', \quad \sigma_{ij} = \langle \sigma_{ij} \rangle + \sigma_{ij}', \dots \quad (3.1)$$

Here, as before, the symbol $\langle \rangle$ denotes the average value of a quantity, averaged over a period of time which is large in comparison with the period of oscillation but small in comparison with the characteristic time of change for the particular property; the prime denotes the oscillatory component. Substituting (3.1) into the basic dynamic equations

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{i\alpha}}{\partial x_\alpha} + g_i^m + g_i^a \cos \omega t \quad (3.2)$$

where (g_i^m, g_i^a) are components of the vectors \mathbf{g}^m , and \mathbf{g}^a and averaging, we obtain

$$\frac{\partial \langle \sigma_{i\alpha} \rangle}{\partial x_\alpha} + g_i^m = 0 \quad (3.3)$$

$$\rho \frac{\partial^2 u_i'}{\partial t^2} = \frac{\partial \sigma'_{i\alpha}}{\partial x_\alpha} + g_i^a \cos \omega t \quad (3.4)$$

Under the influence of a slowly varying quasistatic loading, the stresses and strains within the body are interrelated by means of some rheological relations which depend implicitly on temperature. We may take as an example a nonlinear model generalizing the thermorheological simple model used in [5] for which the following relations between the components of the stress tensor σ_{ij} and the component of the small strains tensor ε_{ij} : hold

$$s_{ij} = \frac{E(T)}{1+\nu} \varepsilon_{ij} - \int_0^t \frac{d}{d\tau} \left\{ \frac{E[T(\tau)] \varepsilon_{ij}(\tau)}{1+\nu} \right\} \Phi_1 [q(t) - q(\tau)] d\tau \quad (3.5)$$

$$\sigma_{\alpha\alpha} = \frac{E(T)}{1-2\nu} [\varepsilon_{\alpha\alpha} - \beta(T - T_0)] - \int_0^t \frac{d}{d\tau} \left\{ \frac{E[T(\tau)] \varepsilon_{\alpha\alpha}(\tau)}{1-2\nu} \right\} \Phi_2 [q(t) - q(\tau)] d\tau$$

or

$$\begin{aligned} e_{ij} &= \frac{1+\nu}{E(T)} \left[s_{ij} + \int_0^t \frac{ds_{ij}(\tau)}{d\tau} \Psi_1 [q(t) - q(\tau)] d\tau \right] \\ \varepsilon_{\alpha\alpha} &= \beta(T - T_0) + \frac{1-2\nu}{E(T)} \left[\sigma_{\alpha\alpha} + \int_0^t \frac{d\sigma_{\alpha\alpha}(\tau)}{d\tau} \Psi_2 [q(t) - q(\tau)] d\tau \right] \end{aligned} \quad (3.6)$$

where

$$\begin{aligned} s_{ij} &= \sigma_{ij} - 1/3 \delta_{ij} \sigma_{\alpha\alpha}, & e_{ij} &= \varepsilon_{ij} - 1/3 \delta_{ij} \varepsilon_{\alpha\alpha} \\ e_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), & q(t) &= \int_0^t F[T(\tau)] d\tau \end{aligned} \quad (3.7)$$

Here F is some function of the temperature, T_0 is the ambient temperature while Φ_1 , Φ_2 , and Ψ_1 , Ψ_2 are displacement and dilatational relaxation and creep functions, respectively; these may, generally speaking, depend on the strain and stress tensor invariants, respectively. In particular, for $\Phi_1 \equiv \Phi_2 \equiv \Psi_1 \equiv \Psi_2 \equiv 0$ we obtain relations for the elastic body, thermal stresses being taken into account

$$s_{ij} = \frac{E(T)}{1+\nu} e_{ij}, \quad \sigma_{\alpha\alpha} = \frac{E(T)}{1-2\nu} [\varepsilon_{\alpha\alpha} - \beta(T - T_0)] \quad (3.8)$$

From the first hypothesis, small vibrations affect the relaxation and creep functions only through changes in temperature, and we can assume that relations (3.5) and (3.6) may be applied to the averaged quantities

$$\begin{aligned} \langle s_{ij} \rangle &= \frac{E(T)}{1+\nu} \langle e_{ij} \rangle - \int_0^t \frac{d}{d\tau} \left\{ \frac{E[T(\tau)] \langle e_{ij}(\tau) \rangle}{1+\nu} \right\} \Phi_1 [q(t) - q(\tau)] d\tau \\ \langle \sigma_{\alpha\alpha} \rangle &= \frac{E(T)}{1-2\nu} \{ \langle \varepsilon_{\alpha\alpha} \rangle - \beta(T - T_0) \} - \\ &\quad - \int_0^t \frac{d}{d\tau} \left\{ \frac{E[T(\tau)] \langle \varepsilon_{\alpha\alpha}(\tau) \rangle}{1-2\nu} \right\} \Phi_2 [q(t) - q(\tau)] d\tau \\ \langle e_{ij} \rangle &= \frac{1+\nu}{E(T)} \left[\langle s_{ij} \rangle + \int_0^t \frac{d\langle s_{ij}(\tau) \rangle}{d\tau} \Psi_1 [q(t) - q(\tau)] d\tau \right] \\ \langle \varepsilon_{\alpha\alpha} \rangle &= \beta(T - T_0) + \frac{1-2\nu}{E(T)} \left[\langle \sigma_{\alpha\alpha} \rangle + \int_0^t \frac{d\langle \sigma_{\alpha\alpha}(\tau) \rangle}{d\tau} \Psi_2 [q(t) - q(\tau)] d\tau \right] \end{aligned} \quad (3.9)$$

Further, in Equations (3.4) the inertia terms are of order $\rho\omega^2 U$ (U is the order of magnitude of the amplitude of vibrational displacements). From the second hypothesis, the oscillatory stresses σ_{ij}' are related to the oscillatory displacements by the constitutive equations of the viscoelastic body. Thus, it may be shown in a manner completely analogous to that of section 2 that the first term in the right-hand side of Equation (3.4) comprises the terms which represent the elastic effect and are of the order $E'(T)U/l^2$ (l is a characteristic length of the body), plus the terms which represent the viscous effect and are of order $E''(T)U/l^2$. Assuming the moduli E' and E'' to have the orders of magnitude given in section 1 and considering the frequencies ω of up to 10^3 sec^{-1} , we may neglect in Equations (3.4) the viscous and inertia effects. Now setting $\sigma_{i\alpha}' = \sigma_{i\alpha}^a \cos \omega t$, $u_i' = u_i^a \cos \omega t$, where the quantities with superscript a (amplitudes) are assumed to vary slowly with time, we obtain for the amplitudes the usual system of equations of the theory of elasticity

$$\frac{\partial \sigma_{i\alpha}^a}{\partial x_\alpha} + g_i^a = 0$$

$$\sigma_{i\alpha}^a = \frac{E'(T)}{2(1+\nu)} \left(\frac{\partial u_i^a}{\partial x_\alpha} + \frac{\partial u_\alpha^a}{\partial x_i} \right) + \frac{E'(T)}{3(1+\nu)} \delta_{i\alpha} \frac{\partial u_\beta^a}{\partial x_\beta} + \frac{1}{3} \delta_{i\alpha} \frac{E'(T)}{1-2\nu} \frac{\partial u_\beta^a}{\partial x_\beta} \quad (3.10)$$

By the second hypothesis, E' is independent of the average stresses. To obtain a complete system of equations, we must have, in addition to (3.10), the energy equation. We shall use the energy equation in the form (cf. [6])

$$\rho c \frac{\partial T}{\partial t} + \frac{\beta T}{3(1-2\nu)} \frac{\partial}{\partial t} E(T) \frac{\partial u_\alpha}{\partial x_\alpha} = \lambda \Delta T + Q \quad (3.11)$$

where ρ , c , λ are, respectively, the density, specific heat and thermal conductivity of the material, while Q is the instantaneous energy dissipation per unit volume per unit time. Since the characteristic time for temperature change is much greater than the period of oscillation, it is possible to write this equation in averaged form. Averaging the dissipation, we obtain $\langle Q \rangle = Q_1 + Q_2$, where Q_1 is the average viscous dissipation of energy based on the averaged deformation, and Q_2 is the average viscous dissipation of energy due to the vibrations which is, as shown in [7], equal to

$$Q_2 = \frac{\omega}{2} I'' \left[(\sigma_{\alpha\beta}^a - \frac{1}{3} \delta_{\alpha\beta} \sigma_{\gamma\gamma}^a) (\sigma_{\alpha\beta}^a - \frac{1}{3} \delta_{\alpha\beta} \sigma_{\gamma\gamma}^a) + \frac{1}{3} (\sigma_{\gamma\gamma}^a)^2 \right] \quad (3.12)$$

where $I''(T, \omega)$ represents the yielding losses discussed in section 2 and is independent of the average stresses, by virtue of the second hypothesis. Estimates show that Q_1 and the average value of the second term in the left hand side are negligible in comparison with Q_2 . Retaining the symbol T for the average temperature, we may write the energy equation as follows

$$\rho c \frac{\partial T}{\partial t} = \lambda \Delta T + \frac{\omega}{2} I'' \left[\left(\sigma_{\alpha\beta}^a - \frac{1}{3} \delta_{\alpha\beta} \sigma_{\gamma\gamma}^a \right) \left(\sigma_{\alpha\beta}^a - \frac{1}{3} \delta_{\alpha\beta} \sigma_{\gamma\gamma}^a \right) + \frac{1}{3} (\sigma_{\gamma\gamma}^a)^2 \right] \quad (3.13)$$

Thus, under the assumptions used, it turns out that, thanks to the elimination from the energy equation of terms which correspond to the average deformations, the system of basic equations separates into two systems: System (3.10) and (3.13) which yields the stress and deformation amplitudes and the temperature, while the average stresses and deformations are obtained from system (3.3) and (3.9), in which the temperature is taken to be a known function of time and position. Once again, we stress that the material model represented by (3.9) is taken for illustrative purposes only and may be replaced by another model.

The boundary conditions for the system (3.3) and (3.9) are

$$\langle \sigma_{i\alpha} \rangle n_\alpha = p_i^m \quad \text{on } S_1, \quad \langle u_i \rangle = u_{0i} \quad \text{on } S_2 \quad (3.14)$$

where n_α are the components of the unit vector normal to the body surface; p_i^m , and u_{0i} are components of the vectors \mathbf{p} , and \mathbf{u}_0 . For equations (3.10) and (3.13), we shall adopt boundary conditions of the form

$$\begin{aligned} \sigma_{i\alpha}^a n_\alpha = p_i^a \quad \text{on } S_1, \quad u_i^a = 0 \quad \text{on } S_2 \\ \partial T / \partial n + \theta (T - T_0) = h \end{aligned} \quad (3.15)$$

Here θ and h are point functions on the body surface; $\partial / \partial n$ is the derivative in the direction normal to the body surface. In addition, we have the initial condition

$$T = T_0 \quad \text{for } t = 0 \quad (3.16)$$

4. We will consider some actual problems which arise in connection with the general investigation developed above.

(1) As a first example, consider the one-dimensional problem of the deformation of a circular cylindrical bar with a constant average load and subjected to a small oscillating load, all properties to be uniformly distributed over the entire cross-section of the bar. Equations (3.9) then take the form

$$\langle \varepsilon \rangle = \beta (T - T_0) + \frac{1}{E(T)} \{ \sigma_0 + \sigma_0 \Psi_1 [q(t), \sigma_0] \} \quad (4.1)$$

$$q(t) = \int_0^t F [T(\tau)] d\tau \quad (4.2)$$

Here σ_0 is the applied, constant, average stress; the function F is assumed to be known. A simplified form of the energy equation should be used. Since the temperature T is also assumed to be averaged over the cross-section of the bar, the resistance to heat conduction must be assumed to be concentrated on the cylindrical part of the surface of the bar. A similar averaging procedure was used by Ratner and Korobov [8] in their initial investigation of the problem concerning the heating of polymers by vibrations in the absence of an average stress field. We obtain

$$\frac{dT}{dt} = \frac{\sigma^{*2} \omega}{2\rho c} I''(T, \omega) - \frac{2\alpha}{\rho c r_0} (T - T_0) \quad (4.3)$$

Here α is the surface conductance for the bar surface, σ^* is the stress amplitude, ω is the frequency and r_0 is the radius of the bar.

The solution of Equation (4.3) with the initial condition (3.16) determines a function $T(t)$ which may be substituted into (4.1) to yield the average deformation as a function of time. The functions Ψ and F are obtained here in the manner similar to that used in a problem [1] in the absence of oscillatory stresses, at various constant temperatures and under the constant applied stress σ_0 . The treatment outlined here differs little from that given in [1].

(2) In investigating the dynamic stability of a body (a detailed development of the problem may be found in the monograph of Bolotin [9]), small oscillating stresses are applied to the body which in the state of elastic stability and the resultant motion is examined. The equilibrium state is either stable or unstable depending on whether or not the displacements arising from this motion remove the body from within a small neighborhood of the undisturbed equilibrium state. However, for polymer materials, even vibrations whose amplitudes are very small in comparison with the original loading may lead to substantial heating and consequent change in the elastic constants. Therefore, a correct study of the dynamic stability of such materials, generally speaking, must be based on a nonlinear formulation developed in the preceding section.

As an example let us consider a circular cylindrical bar which is clamped at the ends and is under a compression load P_0 applied axially, without considering, at present, the parametric resonance phenomena.

Let us apply to the bar an additional oscillatory axial load $P^* = \sigma^* S \cos \omega t$. As before, the amplitude of the oscillatory stress σ^* will be taken as small in comparison with the basic stress σ_0 . The trivial solution (2.1) which describes the linear form of equilibrium of the bar becomes unstable if

$$P_0 > P_c = \frac{4\pi^2 E(T) I}{l^2} \quad (4.4)$$

where l is the length of the bar and I is the moment of inertia of the cross-section of the bar.

The temperature equation will again be taken in the form (4.3). The solution $T(t)$ of Equation (4.3) with initial condition (3.16) is a monotonically increasing function which varies from $T = T_0$ to some value $T = T_1$, where T satisfied the equation

$$\frac{\sigma^{*2} \omega}{2} I''(T_1, \omega) - \frac{2\alpha}{r_0} (T_1 - T_0) = 0 \quad (4.5)$$

As the temperature increases, Young's modulus $E(T)$ decreases, so that at some $T \leq T_1$ the value of $E(T)$ becomes less than $P_0 l^2 / 4\pi^2 I$, then P_0 exceeds the critical load, and buckling takes place, resulting in the loss of stability of the bar. Numerical calculations show that this can occur at oscillatory stress amplitudes which are small in comparison with the basic stress. For example, take a bar whose length $l = 200$ cm and cross-sectional area $S = 100$ cm². Suppose that the moment of inertia of the cross-section of the bar is 800 cm⁴, so that, taking* the modulus of elasticity E at $T = 20^\circ\text{C}$ equal to

* Here and in the following the data for polystyrene has been used for illustrative purposes.

2.5×10^{10} dynes/cm², we obtain a critical force $P_c = 20 t$ at $T = 20^\circ\text{C}$. Let us take an axial load of $P_0 = 10t$, which is half the critical value. If we take the specific conductance as $\alpha = 20$ ergs/cm²-sec°C, then we shall find that at $\sigma^* = 4 \times 10^6$ dynes/cm², or approximately 4 kg/cm², and a frequency $\omega = 10^3$ sec⁻¹ (which corresponds to approximately 150 cycles/sec), the temperature of the bar will increase over a period of time by 100°C, the modulus of elasticity will decrease to half its former value, so that the load will exceed the critical value and the bar will buckle. It should be pointed out that the influence of vibrations increases strongly with the resultant rise in temperature.

(3) In conclusion, we will state some qualitative observations concerning the application of the previously developed approach for the investigation of the influence of vibrations on the strength of polymers.

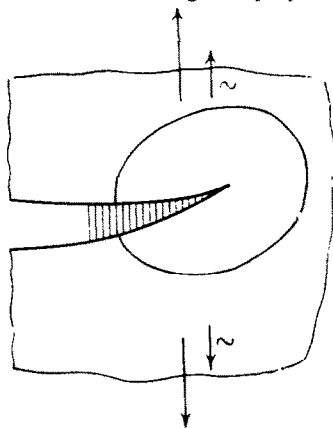


FIG. 3

Polymer materials, like most solid materials contain many cracks (or other stress foci). Small oscillatory stresses produce near the crack ends, oscillatory strains of large amplitudes which result in generation of heat at a higher rate and, because of poor heat conduction, in substantial local increases in temperature near the crack edges. It is known [10, and 11] that opposite edges of cracks in polymer materials are tied together near their ends by thin fibers, the stresses in these fibers playing a fundamental role in providing bonding forces which check the propagation of cracks. (Fig. 3). According to S.N. Zhurkov's kinetic representation, the speed of deformation of these fibers is proportional to

$$\exp\left[-\frac{U - \gamma\sigma}{RT}\right] \quad (4.6)$$

where U is the activation energy, γ is a material constant, σ is the stress in the fiber, R is the universal gas constant and T is the temperature. It is evident from (4.6) that an increase in temperature near the end of a crack, resulting from the vibrations, will greatly alter the speed of deformation of the fibers, thus altering the bonding forces and the bonding modulus. Since such a temperature increase is only local, it may not produce any increase in the rate of deformation of the entire body, but it will change the bond modulus near the end of the crack, thus influencing the strength of the material. The change in material strength resulting from local heating near crack ends in the presence of oscillating loads was first noted by Bartenev [12].

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